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MEASUREMENT OF HIGH ROTATIONAL SPEED*

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(Plate VI)

ABSTRACT. The method is an application of the stroboscopic principle. Intermittent light of very regular frequency is produced by allowing the light reflected from a mirror attached to one prong of an electrically maintained tuning fork to pass through a narrow slit. This light is then reflected from a slightly tilted mirror fixed to the rotating turbine of the centrifuge; as a result, a circle of bright dots appears on a screen placed at a suitable distance. The number of these dots is equal to the denominator of the ratio of the frequency of rotation to that of vibration. To obtain the numerator the slit is moved a little away from the zero-position of vibration of the image of the light source. Each dot splits up into two which again coincide. The displacement of the slit from the zero position necessary to get this coincidence is measured, and from this the numerator is calculated. The method is capable of a very high degree of accuracy.

After the invention of the ultra-centrifuge by Svedberg it became necessary to measure rotational speeds ranging from 500 to 3,000 rev. per sec. Various methods have been suggested from time to time by different workers in the line. Of these a brief account is given below for convenience of comparison. In simplicity, accuracy and range of applicability, however, the stroboscopic method described here seems to have superseded all the others so far adopted for measuring high rotational speed.

(1) The method usually adopted in the earlier work was to view the rotor or turbine of the centrifuge stroboscopically through perforations near the periphery of a rotating disc. The speed of rotation of the disc was maintained at a constant value which was either known or determined by some suitable means. The chief source of inaccuracy in this experiment is that it is almost impossible to keep the speed of rotation of the disc sufficiently uniform for any accurate measurements. Another disadvantage is that with such a stroboscope we can measure only speeds which are integral multiples of the speed of the disc and it is also difficult to know which one of the integral multiples we are measuring.

2. "Recently a more convenient method has been developed by Svedberg, Björnstähl and McFarlane. A portion of the rotor shaft was magnetized lamellarily (by surrounding it with a piece of the same kind of steel during the process of magnetization) and a two-pole stator of a total resistance of 240 ohms and wound with 0.10 mm copper wire was mounted in the centrifuge casing so as to surround this portion of the rotor. When the centrifuge is running an A. C. current of the same frequency as the speed of the rotor is generated." (quoted from "The Ultra-centrifuge" by Svedberg and Pedersen). This current is then amplified and its frequency determined by producing Lissajous figures in the cathode ray oscillograph. The method is accurate and is capable

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of measuring any speed, its only drawback being that the experimental arrangements involved in its use are rather too complicated.

3. Another stroboscopic method due to Harvey (1934) is to illuminate the turbine by means of a neon lamp fed by an intermittent electric current. The interrupted electrical contacts are made by pressing a carbon brush of rectangular section against the rim of a metallic toothed-wheel clamped between two bakelite discs of the same outer diameter. The wheel is made to rotate at a constant speed by means of a synchronous motor. This method is subject to all the disadvantages of the first method.

4. A fourth method due to Snoddy and Beams (1937) is to induce an A. C. current in a coil by fixing a small magnet to the rotor or turbine of the centrifuge. The coil is connected across an A. C. bridge consisting of 4 fixed non-inductive resistances, but in series with one of these is included a variable capacity and a variable inductance which enable the bridge to be balanced at the induced frequency. Though this method is simple the accuracy of the electrical measurements cannot be very high.

For other papers on speed measurement see (1) Pickels (1938) and (2) Bjornståhl (1939).

EXPERIMENTAL ARRANGEMENT

The method is a simple application of the stroboscopic principle. Light from a straight filament lamp was brought to a focus at a distance of about a meter and a half after reflection from a plane mirror attached to one prong of an electrically driven tuning fork of known frequency which was 256 in the

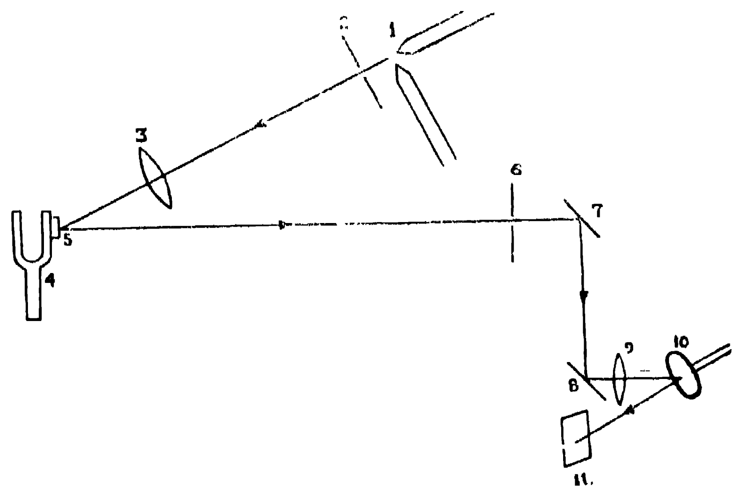

Fig. 1

1. Carbon arc. 2. Narrow horizontal slit. 3. Condensing lens. 4 Electrically maintained tuning fork. 5 Plane mirror attached to a prong of the fork. 6 horizontal slit for producing intermittence of light 7. Movable plane mirror. 8. Fixed plane mirror. 9. Condensing lens, 10. Tilted mirror fixed to the turbine or rotor of the centrifuge, 11. Ground glass screen.

experiment performed (see Fig. 1.) The filament was horizontal and the plane of vibration of the fork vertical so that with the vibration of the fork the image of the filament vibrated transversely. A narrow horizontal slit was then placed so as to coincide exactly with the filament image when the fork did not vibrate. As the fork was then set into vibration the slit allowed light to pass through it only intermittently with a frequency double that of the fork, *i.e.*, $2 \times 256 = 512$ in the present case. A condensing lens was then placed after the slit. Owing to the approximate parallelism of light coming through it this lens condensed the light into a small bright spot on a screen placed at the proper distance. Now a plane mirror was attached to the rotor of the centrifuge such that its normal made a very small angle of a few degrees with the axis of rotation. This mirror was placed after the lens and the light coming through the lens was first reflected from this mirror and then brought to a focus on the screen. With this arrangement when the fork was at rest and the mirror was made to rotate, the spot of light on the screen moved, giving rise to a bright circular ring of small width. This was due to the inclination of the normal to the mirror to its axis of rotation. When, however, the fork was vibrating and at the same time the mirror was made to rotate with slowly accelerated speed, this circular ring broke up into a number of bright dots slowly moving round the circle. This phenomenon was a result of the intermittent nature of light falling on the mirror, and occurred only when the frequency of rotation bore any simple ratio to the frequency of vibration of the fork. For example, when the frequency of rotation was any even integral multiple of 256 only one dot appeared, and when it was any odd integral multiple of 256 the number of dots was two.

THEORY

Let n be the frequency of the fork and $n \times p/q$ the frequency of rotation to be measured, where p/q is a fraction in its lowest terms. The frequency of intermittence is therefore $2n$ and that of rotation can be written equal to $2n \times p/2q$. If p is odd then p and $2q$ are also relatively prime and therefore $2q$ equidistant dots will appear on the circle. If p is even (equal to $2m$, say) then $p/2q = m/q$ and therefore q equidistant dots will appear, where q must be an odd number in this case. So the number of dots gives us complete information about the denominator of the fraction p/q. These remarks are valid if the slit is placed exactly at the zero-position of vibration of the filament image. To determine the numerator of p/q the slit is to be placed a little away from the zero-position, *i.e.*, at a position whose distance from the zero-position measured in phase angle is θ, say. For this eccentric position of the slit the interval between two consecutive flashes will no longer be $1/2n$ but alternately $\frac{1}{2n} - \frac{2\theta}{2\pi}.\frac{1}{n}$ and $\frac{1}{2n} + \frac{2\theta}{2\pi}.\frac{1}{n}$. The 1st, 3rd, 5th,......flashes, *i.e.*, the flashes of odd order will, however, give rise to a system of q equidistant dots, and the 2nd, 4th, 6th...... flashes, *i.e.*, the flashes of even order will give rise to a different system of q equidistant dots on the circle. The two systems may or may not coincide. The angular distance between the two systems is $2\pi n.\frac{p}{q}\left(\frac{1}{2n}-\frac{2\theta}{2\pi}.\frac{1}{n}\right)$. This is equivalent to $2\pi n.\frac{p}{q}\left(\frac{1}{2n}-\frac{2\theta}{2\pi}.\frac{1}{n}\right)-2\pi\frac{k}{q}$ where k is any integer. If p is odd it is equivalent to $2\pi.\frac{1}{2q}-\frac{2p\theta}{q}$ and when p is even it is equivalent to $-\frac{2p\theta}{q}$.

Therefore if the no. of dots is odd (for zero-position of the slit), then each dot will split up into two dots separated by an angular distance $-2p\theta/q$ as soon as the slit is displaced from its zero-position through a small phase-angle θ. If the no. of dots is even no such splitting up of the dots will take place, only the symmetry in the distribution of the dots round the circle will be lost.

The condition of coincidence of the two systems is

$$p\left(\frac{1}{2}-\frac{\theta}{\pi}\right)-k=0 \quad \text{or } p\theta=\pi.\frac{p-2k}{2}$$

If p is odd then this gives

$$p\theta=\ldots\ldots\ldots-\frac{3\pi}{2},\ -\frac{\pi}{2},\ +\frac{\pi}{2},+\frac{3\pi}{2},\ldots\ldots\ldots$$

If p is even then

$$p\theta=\ldots\ldots\ldots-2\pi,\ -\pi,\ 0,\ +\pi,\ +2\pi,\ldots\ldots\ldots$$

It may not be out of place here to answer a question which naturally presents itself. If the frequency ratio be 1/7 001 instead of exactly 1/7 will there be 7001 dots on the circle? The answer will be in the negative. Owing to the failure of the persistence of vision such a large number of dots are never found to appear on the circle. Instead of 7001 dots there will be only 7 dots on the circle rotating with a frequency from which the exact ratio 1/7.001 can be calculated out.

Difficulties in measuring high speed.—The method described above is quite suitable for speeds below 400 r.p.s. A difficulty arising out of the duration of the flashes, however, creeps in as one tries to measure higher and higher speeds. Let a be the width of the filament image focussed on the slit and b the width of the slit and let $w=a+b$. Now if L be the amplitude of vibration of the filament image, then the duration of a flash is $T=\frac{w}{2\pi L}.\frac{1}{n}$ sec. If T is not sufficiently small then for high speeds each dot will be elongated so as to cover a considerable part of the circumference of the circle on which the dots appear. The value of T can be diminished either by decreasing w or by increasing L. As there is a practical limit to the extent to which w can be decreased the only other alternative is to diminish T by increasing L. This has been easily accomplished by attaching two plane mirrors one to each prong of the fork such that they are parallel and their reflecting surfaces face each other. Light from the lamp is now multiply reflected from these two mirrors before coming to a

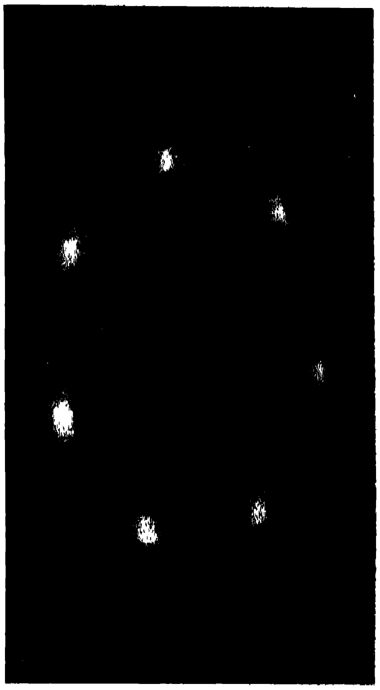

Fig. 2

Photograph showing 7 dots at a speed of 256 × 43/7 = 1572·57 r.p.s.

focus on the slit. If there are r reflections the amplitude of vibration (for a single reflection) is magnified r times. In the actual experiment 8 reflections were taken which gave an amplitude equal to 20 cm. The value of a was 4 mm.

$$\therefore \quad T = \frac{.4}{2\pi \times 20} \cdot \frac{1}{n} = \frac{1}{300} \cdot \frac{1}{n} \quad \text{sec. approx.}$$

Therefore for a rotational speed equal to $10n$ ($=2560$ r.p.s) each dot will cover an arc subtending an angle $360^\circ \times 1/30$. So at the stated speed we will be able to distinguish dots on the circle if their number does not exceed 30. But it should be noted that as one tries to shorten the duration of flashes in this way the intensity of light falls off considerably. This difficulty has been removed by taking instead of an ordinary straight filament lamp a carbon arc as the light source with a water-cooled slit placed very close to it.

PRACTICAL DETERMINATION OF THE NUMERATOR OF p/q

To determine p one is to move the slit away from the zero-position of vibration of the filament image and measure the phase angle θ through which it is moved to get the next coincidence of dots in pairs. If the number of dots is even $p=\pi/2\theta$ and if it is odd $p=\pi/\theta$. Instead of doing this, however, it is more convenient to count the total no. of coincidences that occur as one moves the slit through a fixed phase angle $\pi/4$. From this number N the numerator p can be easily found out. For example, if the no. of dots be odd (for zero-position of the slit) and if there is a coincidence at the position $\pi/4$ of the slit then p is equal to 4 times the no of coincidences observed as the slit is moved from the zero position to the final position, in other words $p=4N$ in this case. It is easy to see how p and N are related to each other in other cases. A difficulty, however, arises as one moves the slit away from the zero position. For if the displacement of the slit is fairly large, the light passing through it will no longer fall on the mirror attached to the rotor. This difficulty has been removed by using two parallel mirrors (Fig. 1) with their reflecting surfaces facing each other, one rigidly fixed to the slit and moving with it and the other kept fixed just in front of the mirror attached to the rotor. Both the mirrors are inclined approximately at 45° to the direction of the incident light which after passing through the slit is now successively reflected from the two parallel mirrors and therefore continues to fall on the mirror attached to the rotor inspite of the motion of the slit. The condensing lens may be placed somewhere between the fixed mirror and the rotor.

DISCUSSION

The advantage of this method is its striking simplicity and the remarkably high degree of accuracy attained by it if proper precautions are taken. A difference of 1/100 cycle per sec. is easy to detect by this method. Great precautions are, however, necessary in order to attain this high order of accuracy. For example, the fork must be of carefully tempered steel and should be maintained preferably by using thermionic valves so that the process of maintenance itself may not affect the vibration of the fork. Finally, the fork should be calibrated by comparison with a standard gravity pendulum at a definite temperature and the necessary temperature corrections applied to the frequency of the fork. Another advantage which is usually lacking in stroboscopic methods is that this method enables one to measure speeds which are not merely integral multiples of the frequency of intermittence but also almost any fractional speed. To measure any fractional speed the only thing required is to devise a suitable means of rapidly counting a large number of dots. The difficulty of counting arises out of the rotation of the dots which are not found to be steady even for a fraction of a second. A method of counting may be to compare the bright dots with a number of opaque dots distributed over the same circle. If, during rotation, the bright dots are at times completely extinguished we can be sure that the no. of bright dots is equal to that of the opaque dots.

The photograph given here (Plate VI) shows 7 dots at a speed of

$$256 \times \frac{44}{7} = 1609.1 \text{ r.p.s.}$$

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REFERENCES

Björnstahl, 1939, *Rev. Sci. Instruments*, **10**, 258-260.

Harvey, 1934, *Biol. Bull.*, **66**, 48-54.

Pickels, 1938, *Rev. Sci. Instruments*, **9**, 354-358

Snoddy and Beams, 1937, *Science*, **85**, 273-274

Svedberg and Pedersen, 1940. The Ultra-centrifuge (Oxford).